Deep Inelastic Scattering

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We have indirect evidence that QCD is the correct theory of strong interactions. It has the correct symmetries that lead to the patterns of meson masses, quantum numbers and decays observed in data.

However, how can we know that QCD is the "correct" theory, among a vast space of possible theories? Moreover, QCD leads to only a qualitative picture of confinement and chiral symm. breaking.

In order to test QCD, we should "see" its fundamental degrees of freedom: quarks and gluons. They are not asymptotic states, but neither is the Z boson. Still, the

Z boson leads to a very particular effect in $e^+e^- \to \mu^+\mu^-$ scattering.

We shall see how quarks and gluons lead to a particular pattern in the final state of high energy collisions.

- $e^+e^- \to$ hadrons

Consider e^+e^- collisions at energies well above 1GeV, but below m_Z so that EW interactions are weak. We define

$$\sigma(e^+e^- \to \text{hadrons}) \equiv \sigma_h(s)$$

as the total cross section to produce <u>any</u> hadronic state ($\pi\pi$, $\pi\pi\pi$, $p\bar{p}$, $K\bar{K}$, $\pi\pi\pi\pi p\bar{p}, \ldots$)

This is an <u>inclusive</u> cross section, as it is the sum of all cross sections of each individual state.

It is unclear how to compute this since for $E \gg 1$GeV, the typical states in

σ_h contain <u>dozens</u> of hadrons in the final state.

Moreover, the chiral lagrangian is an EFT which breaks down for $E \gtrsim 4\pi f_\pi \sim 1\,\mathrm{GeV}$.

The combination of QCD being a weakly coupled gauge theory at high energies and σ_h being an inclusive observable make σ_h calculable.

Let us first relate σ_h to another observable.

Using the optical theorem,

$$-i\left[\mathcal{A}(a\to b) - \mathcal{A}^*(a\to b)\right] = \sum_I \int d\Phi\, \mathcal{A}(a\to I)\,\mathcal{A}^*(b\to I)$$

let's apply it to $e^+e^- \to \mu^+\mu^-$.

Assuming both e^- & μ^- massless, we have

$$\mathcal{A}(e^+e^- \to I) = \mathcal{A}(\mu^+\mu^- \to I)$$

when I are hadronic states. Then, since the total cross section is

$$\sigma(e^+e^- \to I) = \frac{1}{2s}\cdot\int d\Phi\, |\mathcal{M}|^2$$

then the rhs of the optical thm is $2s\,\sigma(e^+e^- \to I)$

Here "I" means asymptotic QCD states, so hadrons, not quarks and gluons.

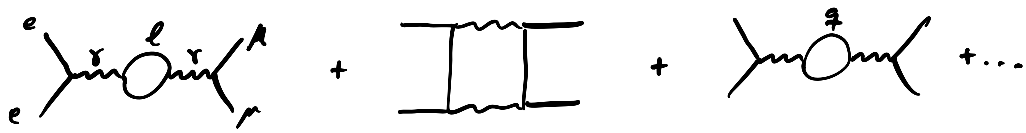
The lhs is the Im part of the $e^+e^- \to \mu^+\mu^-$ amplitude,

$$\sigma_h = \frac{1}{s}\,\mathrm{Im}\big(A(e^+e^- \to \mu^+\mu^-)\big)$$

At tree level,

$$iA = $$ 

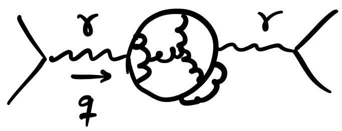
is purely imaginary so $\mathrm{Im}\,A = 0$. At one loop we have



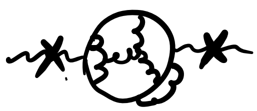
The first two types of diagrams are proportional to the fourth power of the lepton charges. We can take the limit $Q_\ell \to 0$, so the leading contribution comes from the quark loops of the third diagram.

More formally, keeping lepton loops is equivalent to keeping leptonic final states in the total cross section. Including both in the optical theorem doesn't change our conclusions since they can be matched order by order in P.T.

To all orders in α_s, the types of diagrams that enter are



The blob is



$$\equiv i(ie)^2\, \Pi_h^{\mu\nu}(q^2)$$

$$= (ie)^2 \int d^4x\, e^{-iq\cdot x} \langle 0 | T\{ J^\mu_{em}(x)\, J^\nu_{em}(0)\} | 0 \rangle$$

where J^μ_{em} is the QCD electromagnetic current:

$$J^\mu_{em} = \sum_q Q(q)\, \bar{q}\gamma^\mu q .$$

The correlator is evaluated in pure QCD.

The full diagram gives

$$iA_h = (ie)^4 \bar{v}(k_+)\gamma^\mu u(k_-) \frac{-i}{q^2} i\Pi_{\mu\nu} \frac{-i}{q^2} \bar{u}(k_-)\gamma^\nu v(k_+)$$

with $q^2 = (k_+ + k_-)^2 = s$.

Using that J^μ_{em} is conserved,

$$\Pi_h^{\mu\nu} = \Pi_h(q^2)\,(q^\mu q^\nu - q^2 g^{\mu\nu})$$

and using $\bar{v}(k_+)\gamma^\mu u(k_-)\bar{u}(k_-)\gamma_\mu v(k_+) = 2s$ one gets

$$iA_h = i\,2e^4\,\Pi_h(s)$$

which leads to

$$\sigma(e^+e^- \to \text{hadrons}) = \frac{e^4}{s} \operatorname{Im}\Pi_h(s)$$

so the total cross section can be written in terms of the hadronic contribution to the photon self-energy.

The correlator can be computed inserting a complete set of states

$$\Pi(q^2) = \int d^4x\, e^{-iq\cdot x} \langle 0| J^\mu_{em}(x) J^\nu_{em}(0) |0\rangle$$

$$= \underset{\text{hadrons}}{\sum\!\!\!\!\!\!\int} \int d^4x\, e^{-iq\cdot x} \langle 0| J^\mu_{em}(x) |\alpha_{had}\rangle\langle \alpha_{had}| J^\nu_{em} |0\rangle$$

$$= \sum_{\text{had.}} (2\pi)^4 \delta^4(q-p_\alpha) \langle 0 | J^\mu_{em}(0) | \alpha_{had} \rangle \langle \alpha_{had} | J^\nu_{em} | 0 \rangle$$

where $|\alpha_{had}\rangle$ are multiparticle hadronic states.

Now we use the "quark-hadron duality". This is an hypothesis saying that, for inclusive quantities, we can replace the sum over hadronic states with a sum over quarks and gluons. Namely,

$$\sum_{had} \to \sum_{q\bar{q}g}$$

Then,

$$\Pi(q^2) = \sum_{q\bar{q}g} (2\pi)^4 \delta^4(q-p_\alpha) \langle 0 | J^\mu_{em} | \alpha_{q\bar{q}g} \rangle \langle \alpha_{q\bar{q}g} | J^\nu_{em} | 0 \rangle .$$

So instead of computing matrix elements w/ hadrons in the final state, we compute matrix elements of quarks and gluons.

At large enough q^2, we can compute it perturbatively in terms of the strong coupling constant α_s.

At leading order in α_s, this is just the production of a $q\bar{q}$ pair.

Normalizing it to the $\mu^+\mu^-$ cross section, the only difference is the quark charge. Therefore, the ratio is given by

$$R_{had} \equiv \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q (Q(q))^2 + \mathcal{O}(\alpha_s)$$

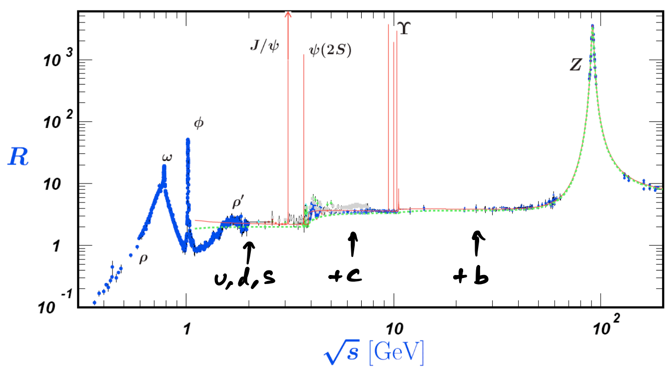
The sum runs over the "active" quarks.

For $E \gtrsim 2\,GeV$,

$$R = N_c\left[\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right] = N_c\,\frac{2}{3}$$

so for $N_c \simeq 3$, $R \simeq 2$. Above the charm threshold, we add $N_c \cdot \frac{4}{9}$, so $R \simeq N_c\,\frac{10}{9}$

Around the $c\bar{c}$ threshold, there are large corrections due to the presence of resonances.

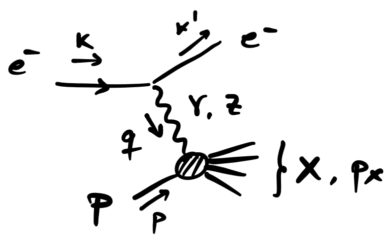


Deep Inelastic Scattering

We can also see the quarks in the so-called Deep Inelastic Scattering experiments. This consists of scattering an electron to a proton at sufficiently high energies. The final state consists of an electron plus a bunch of hadrons

$$e^- p \to e^- X$$

The scattering is mediated by weak interactions. At leading order,



Assuming that $Q^2 \ll m_Z^2$ so we take into account only e.m. interactions, the amplitude is given by

$$i\mathcal{A}(e^- p \to e^- X) = i(ie)^2 \bar{u}(k') \gamma_\mu u(k) \frac{1}{q^2} \langle X | J^\mu_{em}(0) | p \rangle$$

with $p_X = p + k - k' = p + q$.

Given that baryon number is conserved, X should contain at least one baryon. Being the proton the lightest baryon,

$$M_X = \sqrt{p_X^2} \geq \sum_n m_n \geq m_p ,$$

with eq. saturated in elastic scattering.

In general,

$$(p+q)^2 = m_p^2 - Q^2 + 2p\cdot q \geq m_p^2$$

$$\rightarrow \quad Q^2 \leq 2p\cdot q \qquad (q^2 \equiv -Q^2)$$

In the limit of $m_e/Q \to 0$,

$$Q^2 = -(k-k')^2 = 2k\cdot k' = 2E_k E_{k'}(1-\cos\theta_{kk'}) \geq 0 .$$

This suggests to define a dimensionless variable called Bjorken x:

$$x \equiv \frac{Q^2}{2p\cdot q} , \qquad 0 \leq x \leq 1$$

A third variable, besides Q^2 and x, is needed to describe the experiment. We can define

$$y = \frac{p\cdot q}{p\cdot k}$$

In the lab frame, $p^\mu = (M, \vec{0})$ and

$$y = 1 - \frac{E'}{E}$$

where E and E' are the energies of the incoming & outgoing electron.

since $0 < E' < E$, $0 \le y \le 1$

The amplitude contains the matrix element

$$\langle X | J^\mu_{em}(0) | p \rangle$$

which may be a complicated object. However, the differential cross section, inclusive over the hadronic states but as a fn of the e^- momenta,

$$d^3\sigma = \frac{1}{4k\cdot p} \frac{d^3k'}{(2\pi)^3 2E'} \sum_X \int d\Phi_X \frac{1}{4} |A(e^- p \to e^- X)|^2$$

$$= \frac{1}{4k\cdot p} \frac{d^3k'}{(2\pi)^3 2E'} \frac{e^4}{Q^4} \ell^{\mu\nu} W_{\mu\nu}$$

where

$$\ell^{\mu\nu} = \mathrm{tr}(\not{k}'\gamma^\mu \not{k}\gamma^\nu) = 4\left(k^\mu k'^\nu + k^\nu k'^\mu - \frac{Q^2}{2}\eta^{\mu\nu}\right)$$

$$W^{\mu\nu} = \frac{1}{4}\sum_{\substack{h_p \\ (p\text{ spin})}}\sum_X \int d\Phi_X \left(\langle X| J^\mu_{em}|p\rangle\right)^* \left(\langle X| J^\nu_{em}|p\rangle\right)$$

Given that the observable is inclusive on hadronic states, $W^{\mu\nu}$ is a tensor that only depends on the initial proton momentum p^μ, and the final state mom $p_X = p+q$.

Moreover, $W^{\mu\nu}$ is transverse with respect the injected momentum q^μ,

$$0 = \partial_\mu \langle X| J^\mu(x)|p\rangle = \partial_\mu \left(e^{i\overbrace{(p_X - p)}^{q}x}\right)\langle X|J^\mu(0)|p\rangle$$

so

$$q^\mu W^{\mu\nu} = q^\nu W^{\mu\nu} = 0$$

Then, we can write

$$W^{\mu\nu}(p,q) = -F_1\cdot\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)$$
$$+ F_2\cdot\frac{2x}{Q^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)$$

with $F_{1,2}$ being functions of the two scalars made from p, q:

$$q^2 = -Q^2 \quad \text{and} \quad p\cdot q = \frac{Q^2}{2x} .$$

The $F_{1,2}$ are dimensionless. Since n-part states have dim $-n$ and the current is $[J^\mu] = 3$,

$$[\langle X_n | J^\mu | p\rangle] = -n + 3 - 1 = 2 - n$$

The dim. of the phase space is 2 for each particle, so $2n$, plus -4 from the delta fn, so $2n-4$. Therefore

$$[W] = 2n - 4 + 2(2-n) = 0 \quad \longrightarrow \quad [F_{1,2}] = 0$$

By dim. analysis,

$$F_{1,2} = F_{1,2}\left(x, \frac{Q^2}{\Lambda_{QCD}}\right)$$

These are called structure functions. We can express the cross section in terms of them

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\alpha_{em}^2}{xy\,Q^2}\left[\,x y^2 F_1\left(x, \frac{Q^2}{\Lambda_{QCD}}\right) + (1-y) F_2\left(x, \frac{Q^2}{\Lambda_{QCD}}\right)\right]$$

• There is a kinematical regime where a prediction can be made: for elastic ($x \simeq 1$) low virtuality ($Q^2/\Lambda_{QCD} \simeq 0$) regime.

The relevant matrix element is

$$\langle p(p_x) | J^\mu_{em}(0) | p(p) \rangle$$

which can be written as

$$\bar{u}(p_x) \left[A(q^2) \gamma^\mu + i B(q^2) \frac{\sigma^{\mu\nu} q_\nu}{2 m_p} \right] u(p)$$

This is the most general form (recall e^- $g-2$ calculation).

At low virtuality, we also have

$$\langle p_x | Q | p \rangle = +1 \cdot \langle p_x | p \rangle = 2E_p (2\pi)^3 \delta^3(\vec{p} - \vec{p}_x)$$

and

$$\begin{aligned}
\langle p_x | Q | p \rangle &= \int d^3x \, \langle p_x | J^0_{em}(x) | p \rangle \\
&= \int d^3x \, e^{i(p_x - p)\cdot x} \langle p_x | J^0_{em} | p \rangle \\
&= 2E_p (2\pi)^3 \delta^3(\vec{p}_x - \vec{p}) A(0)
\end{aligned}$$

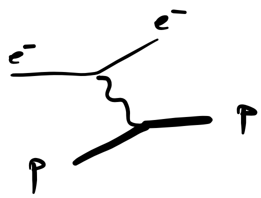
so $A(0) = 1$.

By taking the square of the matrix element, one computes $W^{\mu\nu}$ in terms of A & B.

B only appears with q^2/m_p^2 corrections, so

$$F_1 \simeq \frac{1}{2}\delta(1-x)\underbrace{A^2(0)}_{=1} \qquad F_2 \simeq \delta(1-x)\underbrace{A^2(0)}_{=1}$$

The result is the same as the one obtained by treating the proton as point-like.
The reason is that in this regime the photon has a wavelength $\sim 1/Q$ & does not resolve proton's constituents



- In the deep inelastic regime,
  - $x > 0$ & order 1
  - $Q^2 \gg \Lambda_{QCD}$

is where the structure functions are sensitive to the proton inner structure.
Their measurement lead to the discovery of Bjorken scaling: at large Q^2 but x fixed, the structure functions only depend on x:

$$F_{1,2}\left(x, \frac{Q^2}{\Lambda_{QCD}^2}\right) \simeq F_{1,2}(x)$$

This implies that the theory of strong interactions should be, at least approximately, scale invariant at large Q^2.

Since a class of scale invariant theories are free theories, Feynman postulated that DIS was ocurring by the scattering of a virtual photon with free particles inside the proton, called "parton".

• DIS in the parton model

We can compute the hadronic tensor in the parton model, where the partons are identified with the quarks.

In the parton model, in the rest frame the quarks have momentum $\vec{q}$, with $|\vec{q}| \sim \Lambda_{QCD}$.

In DIS, and in high energy scattering in general, with $\sqrt{s} \gg \Lambda_{QCD}$, we have

$$\overset{e^-}{\longrightarrow} \qquad\qquad \overset{P}{\longleftarrow}$$

$$k \simeq \frac{\sqrt{s}}{2}(1,0,0,1) \qquad\qquad p \simeq \frac{\sqrt{s}}{2}(1,0,0,-1)$$

for the e^- and p momenta. If the quark had energy fraction ξ, in the boosted frame

$$p_q \simeq \frac{\sqrt{s}}{2} \xi (1, 0, 0, -1) = \xi P$$

with small transverse components, of order Λ_{QCD}.

The classical probability to find a quark with energy fraction ξ is $f_q(\xi)$.

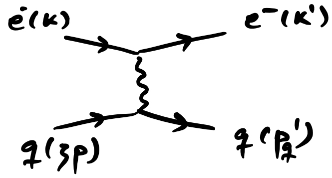
f_q is called parton distribution function and the DIS cross section is given by

$$d^2\sigma\,(e^-(E,\theta)) = \int_0^1 d\xi\, f_q(\xi)\, d^2\hat{\sigma}(e^- q(\xi P) \to e^- q')$$

here $d\hat{\sigma}$ is the partonic cross section, written in terms of partons.

The final state with a single q is because we are working at LO in α_s in the partonic cross section, but notice that, at least formally, f_q is a nonperturbative object.

The partonic amplitude is

$$iA = $$ 

The phase space factor

$$d\Phi_2 = \frac{d^3k'}{(2\pi)^3 2E_{k'}} \frac{d^3p_q'}{(2\pi)^3 2E_{q'}} (2\pi)^4 \delta^4(p_q' - q - \xi p)$$

has to be integrated over $\vec{p}_{q'}$. We can write

$$\frac{d^3 p_{q'}}{(2\pi)^3 2E_{q'}} = \frac{d^4 p_{q'}}{(2\pi)^4} \delta((p_{q'})^2)$$

in order to do the interal with the δ^4. We are left with

$$\delta((p_{q'})^2) = \delta((p+q)^2) = \delta(q^2 + 2\xi p\cdot q)$$

$$= \frac{1}{2p\cdot q} \delta(\xi + \frac{q^2}{2p\cdot q})$$

$$= \frac{x}{Q^2} \delta(\xi - x)$$

So the phase space, at this order, localizes the parton energy fraction ξ to be x, which is related to the outgoing e^- kinematics.

The partonic cross section can be written as

$$d^2\hat{\sigma} = \frac{1}{16 Q^2} \frac{x y}{2\pi} \delta(z - x) \frac{1}{4} \sum_{pols} |A|^2 \, dx \, dy$$

This leads to

$$\frac{d^2\sigma(ep \to eX)}{dx\,dy} = \int dz \, f_q(z) \frac{d^2\hat{\sigma}(z)}{dx\,dy}$$

$$= \frac{4\pi \alpha_{em}^2}{x y Q^2} \left(x y \left(\frac{Q_q^2}{2} f(x) \right) + (1-y) Q_q^2 \, x f(x) \right)$$

So the structure functions in the parton model are

$$F_1(x) = \frac{1}{2} Q_q^2 f_q(x)$$

$$F_2(x) = x Q_q^2 f_q(x)$$

which have the Bjorken scaling.

Moreover, they have the "Callan-Gross" relation

$$F_2 = 2 x F_1$$

which implies that quarks are spin-1/2 particles. This relation would be violated if the quarks were scalars.

Indeed, the hadronic tensor would be

$$\sim \langle \xi p | J^\mu | \xi p + q \rangle \langle \xi p + q | J^\nu | \xi p \rangle$$

$$\sim (2\xi p + q)^\mu (2\xi p + q)^\nu$$

So, after $\xi \to x$, one gets that

$$(2\xi p + q)^\mu \to (2xp + q)^\mu = \left(-\frac{q^2}{p \cdot q} p^\mu + q^\mu\right)$$

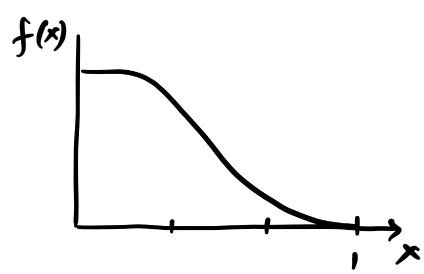
$$\to F_1 = 0$$

for scalar partons.

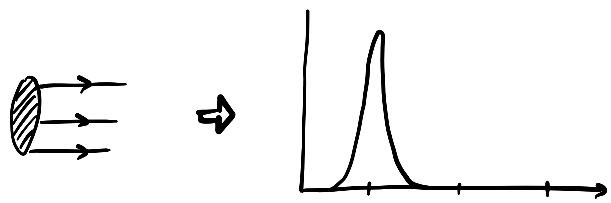
- It is natural to identify the spin-1/2 partons with the quarks.

From DIS experiments one can measure $f_q(\xi)$ and infer properties of the proton constituents.

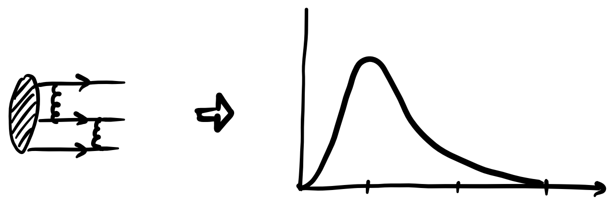
Experimentally, it has the shape



Imagine the proton was made of 3 quarks, so they carry roughly 1/3 of its momentum. In this case $f(x) \sim \delta(x - 1/3)$,



which is clearly not the observed. Now include gluons. This will smear the distribution,



Now add radiation of gluons and pair creation. Like in QED, the spectrum is dk/k or dx/x, which produces a "$q\bar{q}$ sea" at small x: